Approaches to GNSS & Geodesy

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The biggest complaint about state plane coordinates

- The concept that scale and elevation factor need to be applied to the pythagoreum theorem inverse of coordinates to obtain a ground distance confuses/complicates things
- It would be “nice” if grid distance equals ground distance (no factors necessary)
- Grid north does not seem to bother people as it is sort of a well defined assumed north
- Note grid distance not equaling ground distance is often called “distortion” though use of proper mathematics eliminates it

Making grid distance closer to ground distance – approach #1

- Maine Dept. of Transportation has created 3 zones (as opposed to 2) – east, central, and west
- These are not state plane zones, simply zones use by ME DOT for their work
- Scale factor at center of each zone is 1/50000 instead of 1/10000 in Maine East state plane and 1/30000 in Maine West state plane

Making grid distance closer to ground distance – approach #1

- The problem with this approach is elevation factor is still an issue.
- This is because the projections are still defined at the ellipsoid surface

Making grid distance closer to ground distance – approach #2

- County projections – used in Wisconsin and Minnesota
- Each county is its own unique Lambert or Mercator projection
- The projection is at the average elevation of the county instead of at the ellipsoid – elevation factor is computed relative to that average county elevation instead of sea level
- This approach essentially changes the semi-major axis of the ellipsoid by adding the average elevation to it.
- The semi-minor axis is recomputed using the flattening (flattening remains the same)

Making grid distance closer to ground distance – approach #2

- Having the projection at the average elevation of the county minimizes elevation factor “distortion”
- Since the zone only covers one county, which is significantly smaller than a 157 mile wide state plane zone, the scale factor stays much closer to one minimizing scale factor “distortion”

Making grid distance closer to ground distance – approach #2

- In WI and MN in urban areas the combined factor for county projections rarely exceeds 1/100000 and in rural areas rarely exceeds 1/50000
- Thus for most survey applications one can assume a combined factor of one within introducing any significant systematic error if in county coordinates
- Note grid north in county projection will not equal grid north in state plane as different central meridian longitude will exist

Making grid distance closer to ground distance – approach #2

County projection advantages

- Rigidly defined Lambert or Mercator zones

County projection disadvantages

- Can software handle projections not on the ellipsoid?
- Does county projection north not equaling state plane north cause any problems?

Making grid distance closer to ground distance – approach #3

- Called Localization or Localizing coordinates
- One computes one unique average combined average factor (CAF) for a project (average scale * elevation factors)
- Local N = state plane N / CAF
- Local E = state plane E / CAF

Making grid distance closer to ground distance – approach #3

- The survey proceeds in the local coordinate system using plane survey calculations and no scale or elevation factors are required
- If one needs to return to state plane
- State plane N = local N * CAF
- State plane E = local E * CAF
- North in the local system will equal state plane grid north as one overall factor was applied.

Making grid distance closer to ground distance – approach #3

- Two approaches
- AR, NC, WY among others – each project is assigned a CAF at its inception
- TX for example – each district has its own pre-defined CAF and all projects in that district use it

Making grid distance closer to ground distance – approach #3

- Advantages
- Very simply mathematics
- Grid north equals local north

Disadvantages
- Not mathematically rigid definition of a zone
- Need to remember what CAF was used if converting back to state plane
- On large projects, or projects with significant elevation change, “distortion” could become significant

Scale factor change

Maine East Mercator has a scale factor of 0.9999 at its central meridian so it was at the "NAD 27" limit of 1/10000.

The distance from central meridian east (or west) to a scale factor of 1.0001 (1/10000) is 79.3 miles. Note 79.3*2 = 158.6 close to estimate of zone width of 157 miles.

The distance from central meridian east (or west) to a scale factor of 1.0000 is 56.0 miles.

Thus scale change is not linear!!!

Scale change is E-W in Mercator so fits areas elongated N-S better (peninsula of Florida)

Scale factor change

Louisiana offshore is one of the “widest” Lambert zones such that at the central latitude the scale factor is slightly less than 0.9999

The distance from central latitude north (or south) to a scale factor of 1.0001 (1/10000) is 80.0 miles. Note 80*2 = 160 close to estimate of zone width of 157 miles.

The distance from central latitude north (or south) to a scale factor of 1.0000 is 57.4 miles.

Thus scale change is not linear!!!

Scale change is N-S in Lambert so fits areas elongated E-W better (panhandle of Florida)

Low Distortion Projections (LDP’s) – a recent rage

In the 1990’s WI and MN created county projections

Each zone was not on the ellipsoid but instead at the average ellipsoid height/elevation of the county.

Thus a county with an average ellipsoid height of 900 ft. has elevation factor computed by

20906900 / [20906900 + (ellipsoid height – 900)]

Thus elevation factor is “closer” to 1.0000!

The zone is smaller than state plane so the scale factor is always closer to 1.0000 than in the larger state plane zones.

In these initial LDP's semi-major and semi-minor axes of the ellipsoid were "enlarged" by the average ellipsoid height of the county. This changes the value of flattening

f = (a-b) / a because the enlargement cancels in the numerator but not in the denominator.

Several years later this approach to LDP was "discouraged" because

(1)Each county is it own "datum" because a, b, and f have changed

(2)Lots of production software did not have the capability of zones where a, b, and/or f were altered.

Thus in Wisconsin county projections were redefined by the David Allen Coe of surveying math – Alan Vonderohe – to

(1)Work in non-altered a,b,f projections. Mercator stayed the same but Lambert was changed to a one standard parallel projection instead of the state plane two standard parallel approach.

(2)Coordinates in the new system will match the old system to within 5 mm.

Thus a least squares solution was used to “best fit” the new projection parameters to fit the old system.

One parallel Lambert

(1)Central meridian longitude – same as in two parallel

(2)Latitude of one parallel – usually the center of the zone – instead of two latitudes where scale factor = 1

(3)False easting – same as in two parallel

(4)False northing – same as in two parallel

(5)Scale factor at the one parallel latitude – not necessarily one or less than one like in state plane Mercator

Thus less parameters than in the two parallel Lambert of state plane and earlier county projections

LDP’s where one did not have to “back” in to an existing old county coordinate system

Both Oregon and Iowa now have LDP county projection systems.

Latitude and longitude origins are the center of the county.

False Easting and Northing are some logical values to make that county’s coordinates look different than adjoining counties.

Scale factor at origin = 1 / average elevation factor

Thus if average ellipsoid height of county was 2000 ft.
Elev. Factor = 20906000 / 20908000 = 0.999904343
Scale factor at origin = 1 / .999904343 = 1.000095666

Thus combined factor at center of county (the origin) is 1.00000000 !!!!!! Thus grid distance = ground distance.

In reality through trial and error the scale factor at the origin is often shrunk slightly as that makes a CAF of 1 cover more portion of the county as it will be slightly less than one in middle and slightly greater than 1 on edges.

In more reality ellipsoid height change is random across a county so the logic is different in each case.

(1) Dividing coor. by CAF vs. (2) defining a LDP???

(1) Is simpler and only requires remembering a CAF and a spread sheet type calculation.

(1) Preserves state plane grid bearing

(2) Is a true projection (modified state plane)

(2) Is a rigorous application of geodesy

(2) Requires a program to allow and recognize user definition of projection parameters of (Mercator in Main) of

(a) Longitude at central meridian

(b) Latitude origin

(c) False northing

(d) False easting

(e) Scale Factor at central mer. = 1 / elev. factor.

Example – 10 mile area around campus – works great as elevation difference is small (aka cheating in this example)

All results are in meters. LDP's are simple pythagoreum theorem inverse.

	Geodetic	Mercator	Lambert	Geodetic ellip hgt	Geodetic/Merc.
RAY M84	7671.2843	7671.2944	7671.2959	15.23	.999998683
RAY STIL	4633.6295	4633.6217	4633.6214	34.71	1.000001683
RAY 4	3064.5585	3064.5686	3064.5688	2.43	.999996704
RAY THEW	890.4139	890.4157	890.4157	11.93	.999997978
RAY MECC	10529.1828	10529.1952	10529.1963	17.12	.999998822
M84 MECC	16014.2508	16014.2672	16014.2669	18.69	.999998976
4 MECC	7789.6238	7789.6469	7789.6486	5.90	.999997035
4 to MECC is 3 ppm !! RAY to THEW is 2 ppm !!				ave	.999998555

Notice all LDP distances are greater than geodetic except RAY to STIL which is because of STIL's higher ellip. hgt.

Thus you could now "tweek" the scale factor at central meridian to make

the grid distances smaller except that will bring RAY to STIL not as close in comparison.

Lets play with Mercator

change Mercator factor 1.000003686 * .999998555 = 1.000002241 (-446309)

and try again

	Geodetic	Mercator	Geodetic ellip hgt	Geodetic/Merc.
RAY M84	7671.2843	7671.2834	15.23	1.000000117
RAY STIL	4633.6295	4633.6149	34.71	1.000003151
RAY 4	3064.5585	3064.5642	2.43	.999998140
RAY THEW	890.4139	890.4144	11.93	.999999438
RAY MECC	10529.1828	10529.1800	17.12	1.000000266
M84 MECC	16014.2508	16014.2440	18.69	1.000000425
4 MECC	7789.6238	7789.6356	5.90	.999998485

ave 1.000000003 wow!!

Next I averaged all Maine East Mercator scale factor to get .9999026580

and averaged all ellipsoid heights to get 17.046 m = 55.93 ft and obtained

an elevation factor of 0.999997325 to obtain a CAF of 0.999899983 which

was used to localize the state plane coordinates.

	Geodetic	Localized	Geodetic ellip hgt	Geodetic/Localized
RAY M84	7671.2843	7671.2848	15.23	.999999935
RAY STIL	4633.6295	4633.6168	34.71	1.000002741
RAY 4	3064.5585	3064.5639	2.43	.999998238
RAY THEW	890.4139	890.4142	11.93	.999999663
RAY MECC	10529.1828	10529.1885	17.12	.999999459
M84 MECC	16014.2508	16014.2634	18.69	.999999213
4 MECC	7789.6238	7789.6430	5.90	.999997535
			ave	.999999541 wow again!!

The simple averaging tends to give the weight to areas with more points and obviously if the CAF was now tweeked by 1/.999999541 things would get closer.

Be careful

If overlaying on orthophotos you have to convert your coordinates back to the projection of the orthophotos.

Document the CAF or LDP parameters in a survey report or on a plan.

I wish our coordinates were latitude longitude and coordinate geometry would work with it but that is not the CAD world we live in.

But note a lot of CORS stations really really helps RTK GPS!

A lot of RTK GPS also really really helps VRS (Virtual Reference Station) which has been available in many parts of the country for free or a very realistic “use” fee

WE HAVE COMPLETED THE CUSTOM GEOID FILE FOR THE STATE OF FLORIDA.

The Florida Custom Geoid Model, "FPRNGD16", expands on the NGS “Geoid12b” in two ways:

(1) The number of points used by FPRNGD16 was increased by nearly 50 percent over that used by Geoid12b. Geoid12B uses 2470 points in its hybrid model while FPRNGD16 has added 1002 points to the model.

(2) Geoid12b does not fit used points exactly, FDOT has improved to make the hybrid model fit closer.

In the FPRNGD16 model it was decided to use 0.08 meters as a tolerance where GPS benchmarks would be rejected - this resulted in 118 of the 3472 total points being removed.

The FPRNGD16 hybrid geoid model has a root-mean-square error (RMS) of fit for the GPS benchmarks of 0.008 meters. The corresponding RMS of the Geoid12b hybrid model is 0.022 meters (both computed after excluding those misfits larger than 0.08 meters).

The FPRNGD16 hybrid geoid model has 3025 GPS benchmarks that fit better than 0.01 meters. The corresponding Geoid12b hybrid model has 1712 points which fit better than 0.01 meters.

The FPRN geoid model is of NGS standard "bin" format gridded to the NGS standard of one minute of latitude and longitude.

Surveyors should have the opportunity to create their own geoid models or contribute to local data banks that are maintained by local or state agencies.

Questions?????